An Efficient Markov Chain Model Development based Prefetching in Location Based Services

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https://doi.org/10.56343/STET.116.014.001.006

http://stetjournals.com

Article History Received: 03-03-2018 Revised and Accepted : 07-08-2020 Published: 26-09-2020

Abstract

A quite significant issue with the current Location based services application is to securely store information for users on the network in order to quickly access the data items. One way to do this is to store data items that have a high likelihood of subsequent request. This strategy is known as proactive caching or prefetching. It is a technique in which selected information is cached before it is actually needed. In comparison, past constructive caching strategies shown high data overhead in terms of computing costs. Therefore, with the use of Markov Chain Model, the aim of this work is to address the above problems by an efficient user future position movement prediction strategy. For modeling of the proposed system to evaluate the feasibility of accessing information on the network for location-based applications, the client-server queuing model is being used here in this chapter. In contrast observational findings indicate substantial improvements in caching efficiency to previous caching policies that did not use prefetch module.

Key Words: applications and services, content caching, context-aware mobility, context prediction, Location Based Services, mobile environment, proactive caching, prefetching

INTRODUCTION

In the modern age, access to the information is quite easy (Ben *et al.*, 2017)(Ajay Kr Gupta and Shanker, 2020b)(A. K. Gupta and Shanker, 2020). Users can quickly access data stored in videos, images, and sounds, enabling for internet service providers to serve in a better way. Proactive caching is a well-known methodology for performance optimization that significantly improves data look-ups. The past research claimed that latency could be greatly decreased by embedding proactive caching

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email: *prabha200481@yahoo.co.in* Technical Test Lead, Infosys Limited, Chennai. phenomenon. The approach helps the consumer to have easy access to the data without having to request from server and reduces unnecessary overhead. Proactive caching focuses on the user's ability to anticipate mobility trends by travelling from one place to another in an environment in order to assess which data objects should be placed on which cache node, depending on the frequency of content demanded by customers. The traditional prefetching strategies overcome the latency state of the processor only; the encouraging aim of the prefetching approach to be used against mobile devices, however, is to minimize both the time of processing as well as time for communication (Gupta and Shanker, 2020) (Gupta and Shanker, 2020). The problem statement for prefetching is also to suggest a new approach to reinforce the prefetch processing of a low network communication mobile device system (Gupta and Prakash, 2018). With the support of machine learning techniques, this approach improves customer satisfaction and optimizes cache access. Prefetching has been reported in recent studies to be a hot subject for research on computer-based web services. Informed mobile prefetching (IMP) architecture had been designed in (Patterson et al., 1995) to prefetch data using least recently used (LRU) cache, which utilizes costs and benefits analysis to handle the allocation of the disk buffer to competitive users. Previous prefetching research was built specifically for desktop environments, and is agnostic to resource limitations imposed by CPU, memory and mobile device battery power (Gupta and Shanker, 2018b). Thus, since the main aim of prefetching was to minimize access latency, all of the prefetching logic might historically operate on the end computer. However, a smartphone and tablet prefetching approach needs to be built to minimize the demand on the system resources as well as the remote access. Therefore this chapter presents the prefetching process that may be appropriate for content caching in the mobile world

by looking at this promising solution. This chapter's key contribution is to make a model to decide the data objects to be prefetch using the Markov model.

Preliminary

This subsection have the definition of the terms used in this paper.

Definition 1: Road Network- The road network is defined by a undirected graph G=(V, E), comprising set of road segments represented as E and set of intersections represented as V.

Definition 2: Location- It is the recorded position of given moving object in terms of latitude and longitude coordinates in two dimensional area of interest.

$$P\{M_{n+1} = j | M_0, \dots, M_n\} = P\{M_{n+1} = j | M_n\}$$
$$P\{M_{n+1} = j | M_n = i\} = p_{ij}$$

Definition 3: Markov chain-Let S be the set of all states, then the sequence $\{M_t, 0 \ d \le t\}$ is said to be a Markov chain if for any i, j S and $0 \le t$ following condition hold.

The p_{ij} is the probability of transition to jth state from ith state. The matrix $P = (p_{ij})$ is the transition matrix of the chain and the transition probabilities satisfy $\sum_{j \in S} p_{ij} = 1$, where $i \in S$. This means that the likelihood that it will make a transition to another jth state depends only on the ith state regardless of its past prior to time n (Pitman and Yor, 2018). It should be remembered here that it does not matter if the particle was in that state for just a fleeting moment or a lengthy period of time. Table 1 describes the various terminologies used in the chapter.

Problem Statement, Motivation and Summary of Contributions

In order to find the solution of real-world problems, there are number of well-known prediction approaches; examples of such predictive models include the ANN-Support Vector Machine (SVM), Empirical Mode Decomposition (SVR-Hybrid, & IMFs-Hybrid) (EMD-SVR-Hybrid), Autoregressive Moving Average Model (ARIMA), Seasonal ARIMA(SARIMA), Artificial Neural Networks (ANN), Fuzzy Time Series (FTS) (Ajay Kr Gupta and Shanker, 2020a). Each model of prediction has its own strengths and expertise in tackling different problems in the real world. The Markov chain is one of the influential techniques developed to solve complex real-world problems, such as model interrelationships, peak energy consumption estimation etc.

A special case of the stochastic process is the Markov chain which was termed after the Russian mathematician Andrei Andreevich Markov (Ruiz Espejo, 2002). This Methodology have recently been used to approximate the transitive matrix from the system observation states. It is a random mechanism in which the current state records all information about the future. Furthermore, state transformation matrix and probability are the key components in designing the Markov chain model; both of which would summaries all the critical dynamic transformation parameters.

Through incorporating a revised solution to the unique next position prediction system and the cost role of data items capable of addressing LBS issues, the research challenge is to improve the cache hit ratio of

Symbol	Description	Symbol	Description
0	Outlier	Zaccess	Zipf access distribution parameter
Size_Rect	Size of the service area	Num_Scope	At different locations, the number of data items single values
n.	Count of states	М	Cells count
Query_Interval	Subsequent query average time interval	А	Latest estimation access probability biasing constant
POI_Num.	Point of Interest Count	confmin	Minimum confidence threshold
N	Trajectories dataset count (number of users)	Query_Interval	Subsequent query average time interval
Band_Range	Bandwidth Range	Moving Interval	Predicted Region Computation Time interval
S _{min}	The minimum size of the data item	G	Road network undirected graph
C_Size_Ratio	Cache vs. database size	S	Set of all states

Table 1. Terminology Description

the successful caching strategy (Gupta, and Shanker, 2019) (Gupta, and Shanker, 2018a). The paper's problem description can be described as below.

Problem Definition- Let $T = {T_1, T_2, ..., T_N}$ be the N trajectories containing historical trajectories from database. Then, the goal is to find the set of maximum data items that can be stored in prefetch cache storage to maximise the cache hit rate. To predict the set of data items, system finds the probable future location L_i through markov chain model. The major contributions of this work are outlined in the points given below.

1. Proposal of a revised approach to improve the precision of the next location prediction procedure for designing of efficient prefetching method and reduce the processing overhead - Using the user's movement trajectory, the process of next location prediction through the mobility Markov chain model has the better accuracy and least error than that of previous forecasting methods used in caching.

2. Proposal of a revised data item's cost function using the distance between the client's estimated next locations to the reference point of a valid scope - It contributes to the cache hit ratio improvement using prefetching.

3. Analysis of efficiency and overhead of proposed Caching scheme compared to the previous scheme.

The composition of the chapter is as follows. The proposed methods are defined in section 2. Section 3 consequently illustrates the design and study of the simulation. Finally, the chapter is concluded in section 4 with the list of scopes for potential LBS_study.

Proposed Methods

$$P\{M_{n+1} = j | M_0, \dots, M_n\} = P\{M_{n+1} = j | M_n\}$$
$$P\{M_{n+1} = j | M_n = i\} = p_{ij}$$

In this research, the Markov chain technique (Gupta and Shanker, 2020) was adopted. Through incorporating the simplest method of statistical depende $\sum_{j \in S} p_{ij} = 1$ proposed method is a stochastic operation. Furthermore, the future functioning of this model is not concerned with previous behavior. Let S be the set of all states, then the sequence $\{M_t, 0 \le t\}$ is said to be a Markov chain if for any i, j S and $0 \le t$ following condition hold.

The p_{ij} is the Markov chain transition probability form i^{th} state to j^{th} state. Here, the matrix $P = (p_{ij})$ be the transition matrix of the chain, and matrix probabilities satisfy, where $i \in S$. The proposed Markov chain process algorithms consisted of five steps that is being shown in Fig 1.

$$\mathbf{N} = \begin{vmatrix} n_{11} & \dots & \dots & n_{1s} \\ \vdots & \vdots & \vdots & \vdots \\ n_{s1} & \dots & \dots & n_{ss} \end{vmatrix}$$

Step 1. Defining the Markov Chain Process Condition - In this phase for the Markov chain method, thresholds or states must be calculated on the basis of the data used in the model growth.

$$\mathbf{P} = \begin{vmatrix} p_{11} & \dots & p_{1s} \\ \vdots & \vdots & \vdots \\ p_{s1} & \dots & p_{ss} \end{vmatrix}, \, \mathbf{i}, \, \mathbf{j} \in \mathbf{I},$$

Step 2. Build the matrix of state transition, N, and

$$P\{M_{n+1} = j | M_n = i\} = p_{ij}$$

probability of state transition- As defined by the Markov chain, the transition matrix N of state indicates the observed frequency of state transition. Therefore,

Where transitions count of state i to state j is depicted by n_{ii} . F or each state of the Markov chain model we



Fig. 1. Steps in Markov chain Model

assumed P be the transition matrix which represents all the concerned transition probabilities. P is therefore denoted as below,

Then,

The above terms are used for one-step probability of homogenous or stationary Markov chain. The transition probabilities in this markov chain are independent of time t. The markov probability is the kstep transformation probability from state i to state j. The following property is prevalent in the transition matrix P.

 $P(n) = P \times P^{n-1} = P^n$

Step 3. The ergodic properties confirmation in Markov chain- In order to define the presence of restrictive distribution in this chain by classifying the condition of P, proof of an ergodic Markov chain must be made. The existence of aperiodic states and positive recurrent

$$\sum_{n=1}^{\infty} P_{ii}{}^{(n)} = \infty$$

are concluded as ergodic. The ergodic properties can be classified into three pieces;

1. Irreducible-. The Markov chain could be deduced as irreducible when there is just one class and the two states communicate in this same class.

2. Periodicity- Let d is the largest integer, and n is not divisible by d, then d is said to be period of State i. In this case . The Markov Chain is known as aperiodic is its each state has a period of one.

3. Transient and Recurrent States- Assume f_i be the probability of re-entering the given state from its starting state i, then state i is said to be transient if $f_i < 1$ and recurrent if $f_i = 1$. The recurrent condition for finite Markov chain is met only in below case.

Step 4. Probability values computation for Markov process- The mean return time and Stationary probability $\pi_j = \lim_{n \to \infty} P\{M_n = j | M_0 = i\}$ in be obtained f $\sum_{n \to \infty} P\{M_n = j | M_0 = i\}$ in ting, station $P_j(n) = \sum_k P_k (n-1)P_{kj}$ bution can $\sum_k \pi_k P_{kj}$ e the $\mathbb{R} \subset \mathbb{R}^{k}$ for of air emissions where the chain is appropriate for a long period of time with stable-state probability which do not depends on of starting conditions (Trivedi, 2016). The restrictive distribution existed π_j for the stationary distribution of probability for an ergodic Markov chain and can be defined as



Therefore, the likelihood of having the process in state j is independent of the starting condition over a long

$$P(S_j) = \sum_{i=1}^{j} P(S_i) P_{ij}$$

period of time in the process. If the likelihood of the incidence of state j is high, the value of π_j will be more (Grinstead and Snell, 2021). In addition, it is important to measure the mean return time to identify the average time for particular states to return to itself, mj. It can be given by $m_{ij} = 1 / \pi_j$.

$$\chi^2_{\text{calculated}} = \sum \frac{(\text{Expected} - \text{Observed})^2}{\text{Expected}}$$

Step 5. Forecasting and model validation f^{2} The forecast measure can h^{2} derived from the below equation using the initial likelihood and probability of state transition.

Where P_{ij} is a probability of state transition and P(Si) is an initial probability. The Chi-square test is used for model validation to verify the validity of the Markov chain based on the independence assumption (Zhou *et al.*, 2018). The alternate hypothesis is that the consecutive time data selected is dependent, while the null hypothesis is that the consecutive time data selected is independent.

$$\max(\sum_{\mathbf{d} \in S} \mathsf{cost}(\mathbf{i})) \, \mathsf{And} \ \sum_{d_i \in S} \mathsf{objSize}(d_i) > Total \, Prefetch \, Occupied \, Cache \, Size$$

The null hypothesis is rejected when is greater than on the 0.05 critical regions (Zhou et al., 2018).

A proactive caching framework focused on the mobility paradigm known as EMC-prefetching is being suggested in this chapter. EMC-prefetching method has been proposed to estimate future movement routes, $P_i, A(vs(a_i)) \lambda_i$

$$Cost_{i} = \begin{cases} \overline{D'(vs(d_{i}))}, \overline{S_{i}}, \overline{\mu_{i}} & \text{if } vs(d_{i}) \notin pred_Reg \\ \frac{1}{\min(L_{r}, D(vs(d_{i})))}, \frac{P_{i}.A(vs(d_{i}))}{S_{i}}, \frac{\lambda_{i}}{\mu_{i}} & \text{if } vs(d_{i}) \in pred_Reg \end{cases}$$

which is included in the revised cost feature to increase the cache impact ratio. Prefetching is performed to keep the hot data items in cache for subsequent request. The proposed policy is intended to identify the cache data items having the maximum cost for data item.

$$P_i = \frac{1-\alpha}{P_i} + \frac{\alpha}{t_c - t_i^l}$$

The formal definition of the rprefetching function can be given as below.

38

If the valid scope of data items has a larger area, then the probability of revisiting it in the region is greater. The different input parameters in the estimation of data item expense for cache replacement are the distance from the relevant scope to the actual client location, data item size, valid scope, and probability of the access.

The access possibility is represented by P_i and has zero as its initial value. The area of valid scope is represented by .Let, the last access time to the ith data element is t_i^1 , then the update function of access likelihood can be described by the equation given below.

The next position anticipation algorithm is used for finding selecting the data items for prefetching in the

$$D(vs(d_i)) = |(L_{am} - L_i)| = \sqrt{(Ly_{am} - Ly_i)^2 + (Lx_{am} - Lx_i)^2}$$

cache. The customer next position estimation integrated with Markov chain matrix-based procedure $D'(vs(d_i))$.

$$D'(vs(d_i)) = |L_p - L_i| \sqrt{(Ly_p - Ly_i)^2 + (Lx_p - Lx_i)^2}$$

is described in the earlier section. Using the next predicted location, the distance from the client's next predicted location to the reference point of the data object's valid scope may be defined. The mobility regulations were interpreted through the Markov mobility matrix depending upon the similarity between the users' movement logs. The approach effectively increases the cache hit ratio relative to earlier policies due to the specific integration of the next location prediction feature in data item cost computation. The distance between reference point $L_i = (Lx_{i'}Ly_i)$ for the valid scope of ith data item and client's estimated next locations $L_{am} = (Lx_{am'}Ly_{am})$ for the next query data item (d_i) is given by defined as below.

The distance between the predicted region center to the reference point of valid scope is expressed by .

On client m query issue time, $V_{m'}L_m = (L_{xm'}L_{ym})$ and $L_i = (L_{xi'}, L_{yi})$ is the velocity, current position, and reference point of valid scope are respectively. The average update rate and average query rate are defined by μ_i and λ_i respectively for the ith data set. In this strategy, the revised cost function is proposed that enhances the temporal locality functionality by query rate to update the rate ratio.

EVALUATION OF PROPOSED MODEL

The evaluation was conducted on Octa-core 3.2 GHz, RAM of 64 GB, Windows 8 operating system, and Intel i7 processor. The processing time overhead of the query and service schedule is assumed to be negligible in the proposed model. Location-based services have



a. 110 data items



b. 220 data items

Fig. 2. Scope Distribution

drawn millions of users and their digital footprints are massively contained. The query process and interval process are the two modules executed for the simulation of the proposed model.

Data Collection and API States

The 10 month's duration data of foursquare check-ins in New York is used to evaluate the feasibility of our proposed approach and the data availability. The cumulative points count is about 227,428 check-ins with each check-in is associated with 8 attributes. A location-dependent k-nearest neighbor query (e.g. nearest hospital profile info) is continuously generated by the query process with the exponential distributed query interval. In Figure 10, the Voronoi diagrams for 110 and 220 data objects are represented. We used Fortune's algorithm to create a Voronoi diagram from a series of points in a plane. The Fortune algorithm uses the logical surface of the sweep to solve various challenges of Euclidean space.

$$\operatorname{Zipf}_{\operatorname{prob}}(i) = \frac{\frac{1}{i^{Z_{access}}}}{\sum_{j=1}^{U} \left(\frac{1}{j^{Z_{access}}}\right)}$$

In the Foursquare data set, an extra attribute namely queried data item is also added. This attribute defines the details of the data item associated with a given check-in location using the corresponding valid scope of the Voronoi diagram.



Fig. 3. Cache Replacement Schemes without Prefetching Procedure







Fig. 4. Cache Replacement Schemes with EMC-Prefetching Procedure

For performance analysis of the model, the effects of variation in queries inter-arrival time and cache size are plotted with the cache hit ratio. The interval process is also exponential distributed for the computation of predicted regions. The access pattern for data items follows a Zipf distribution to simulate non-uniform distribution for access to a data item. The data items are arranged in such a way that 0th data item be the most frequently accessed and (DB_{Size} – 1)th data item be the least frequently accessed. The Zipf probability equation given below is used for ith data item access probability.

Where the total count of data items and Zipf ratio are represented by U and Z_{access} respectively. In the case of $Z_{\text{access}} = 0$, the uniform access pattern with common likelihood is used for every data item. Skewness in access pattern has been shown with the increasing value of Z_{access} .

DISCUSSION

In this subsection we discuss the simulation results from the application of the Proactive caching strategy. For simulation of proposed strategies, we use Matlab software. Here we have compared the various existing caching policies such as LRU (Xu *et al.*, 2004), FAR (Ren and Dunham, 2000), MPRRP (Gupta and Shanker, 2018c) and SPMC-CRP (Gupta and Shanker, 2018d) content caching without prefetch and then we use with integration of proposed EMC-prefetch based model.

CONCLUSION

We analysed the issue of current prefetching strategies from various perspectives in this article, namely prediction functions, access methods, and uncertainty. Compared to previous prefetch-based caching policies for LBS, the projected area computation-based location-dependent data prefetching performed as described in the proposed policy achieves substantial cache hit ratio performance improvement. We concluded that it is essential for the database researchers to think to offer scalable, generic, and complete caching solutions over Spatio-temporal data for mobile objects feasible to be combined with traditional DBMSs. We have also addressed key research challenges that researchers should take into consideration while planning to construct solutions to major research problems. In nutshell, the target strategy is to devise a way to provide a forecast feature that may correctly work for short-term prediction as well as longterm prediction.

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